Kinetic uncertainty relations in stochastic control

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Non-equilibrium stochastic reaction networks are commonly found in both biological and non-biological systems, but have remained hard to analyze because small differences in rate functions or topology can change the dynamics drastically. Here we conjecture exact quantitative inequalities that relate the extent of fluctuations in connected components, for various network topologies. Specifically, we find that regardless of how two components affect each other's production rates, it is impossible to suppress fluctuations below the uncontrolled equivalents for both components: one must increase its fluctuations for the other to be suppressed. For systems in which components control each other in ring-like structures, it appears that fluctuations can only be suppressed in one component if all other components instead increase fluctuations, compared to the case without control. Even the general N-component system—with arbitrary connections and parameters—must have at least one component with increased fluctuations to reduce fluctuations in others. In connected reaction networks it thus appears impossible to reduce the statistical uncertainty in all components, regardless of the control mechanisms or energy dissipation.